The Cosmological Flatness Problem and Limits on Dark Matter

George W. Collins, II

The Warner and Swasey Observatory of Case Western Reserve University

10900 Euclid Av., Cleveland, Ohio 44106-7215, USA

To be published in *Comments on Astrophysics* Vol. 18, No. 6.

Abstract

The traditional argument to justify $\Omega_0 = (\rho/\rho_{crit}) = 1$ which suggests an absurdly close agreement between the initial expansion energy of the universe and its binding energy is reversed to show that it is unreasonable to expect the current value of W to be within a factor of ten or more of unity. The Uncertainty Principle is applied to the initial value of the mass require to close the universe. The tiny fractional uncertainty is amplified so that the current fractional uncertainty is of the order of unity. This suggests that the requirement of exact initial agreement between the binding energy and the expansion energy is unreasonable as it can never be tested by contemporary measurement.

Nearly thirty years ago Robert Dicke[3] introduced the notion that the universe must be "extremely finely tuned" to yield the present observed balance between the energy of expansion and the gravitational self binding energy. That balance is usually described in terms of the ratio of the current local matter density to that required to close the universe [i.e. $\Omega_0 = (\rho_0/\rho_{crit}) = 1$]. While present observational limits on Ω_0 are probably generously described as $0.1 < \Omega_0 < 10$, such an apparently wide range suggests a very much narrower range at earlier epochs. This results basically from the non-linearity of the dynamical laws governing the expansion. During the vast majority of the time since the origin of the universe the laws that govern the expansion are adequately characterized by the those of Newton. Dicke suggests that the present range of W would have to be decreased to one part in a thousand at the time galaxies began to form and by 1 part in 10^{13} during the age of nuclear reactions when the elements re-formed. A further reduction is required as one moves back further in time toward the origin of the universe. If one were to push this argument through the inflation era to the earliest epoch of the universe one would find as did Lightman and Gringerich (1992), that Ω can depart from unity by only one part in 10^{59}. The continual approach of W toward unity as one moves back through the universe through eras of physical structure having increasing uncertainty has been used by many to assert that $\Omega_\mathrm{i} = 1$ is a initial condition of the structure of the universe since any "real" departure from unity as small as 1 part in 10^{59} is tasteless. This argument has become known as the Flatness Problem.

In this essay, I take an alternate view. I argue that the proper view of the universe is to follow its evolution from the initial conditions to the present and what results is what should be expected. However, of greatest importance is the notion that any observable quantity can have an exact value. One of the triumphs of the twentieth century is that the exactness of nineteenth century classical physics has been replaced with a quantum uncertainty described by Heisenberg's interpretation of Quantum Mechanics. While such uncertainty in, say, the binding energy of the universe is likely to be ridiculously small, we have just seen that ridiculously small departures of Ω_i from unity can lead to substantial departures in the present era. Thus, the very argument used to force the value of Ω_i toward unity should be inverted to show that tiny quantum uncertainties in that observable parameter evolve into significant uncertainties in the present. So we are not allowed to use the "Flatness Problem" to force perfection on the initial conditions of the early universe and then invert the procedure and demand that $\Omega_0 = 1$ in the current era. Indeed, we should be very surprised if Ω_i turned out to be exactly unity. Let us take a moment to see how the Flatness Problem arose. Then we will attempt to estimate the impact on the current value of Ω_0 of Heisenberg Uncertainty Principle applied energy of the universe at the earliest time allowed by contemporary physics.

Ever since the interpretation of the large red-shifts of distant galaxies found by V.M. Slipher (e.g. Osterbrock[10]) as a general expansion of the universe by Edwin Hubble[6,7], astronomers have used the physical laws of the present to describe the images of the past as seen in their telescopes. The discovery of radiation from the primeval fireball by Penzias and Wilson[12] confirmed an earlier prediction by Alpher and Herman[1,2], inspired by Gamow[4], that was based on extrapolating the physics of the present far into the early stages of the past. This venerable approach to understanding the early phases of the universe continued to push back the era of understanding through a time when nuclear physics dominated the world (e.g. Weinberg[13]).

The reversing of the observed expansion of the universe so as to interpret the early structure of the universe requires that the earliest phases consist of matter at extreme temperatures and densities originating in what has come to be called the"Big Bang". Two fundamental problems with the Big Bang cosmology emerged in the sixties and were called the Horizon Problem and the Flatness Problem. The former arises from the similarity of conditions in different parts of the primeval fireball which could have never been in contact with each other throughout the million years or so during which they traveled to the points in space-time where they released the photons observed today. Such similarity could only be established by mutual interactions which could never have taken place. When they emitted the photons we see today they simply existed beyond each other's current event horizon. Alan Guth (1981) suggested a modification to the standard Big Bang Cosmology called *inflation* wherein an extremely rapid expansion took place during the earliest phases of the universe which allowed those widely separated places to once have been in contact and thereby to adopt similar characteristics.

The "Flatness Problem" is somewhat different in concept. The dynamical Big Bang Cosmology has three possible outcomes depending on initial conditions. If the gravitational attraction of the matter in the universe is strong enough, it will eventually stop the expansion and cause the universe to collapse to a point in an event sometimes described as the "Big Crunch". On the other hand, should the initial bang have been sufficiently intense then the expansion energy will overwhelm the self-gravity of the matter and the universe will expand forever. Clearly there is a special case between these two where the gravitational self-energy will exactly equal the expansion energy and the expansion will stop after an arbitrarily large time has elapsed. In the language of General Relativity these three cases correspond to the space-time of the universe being positively curved, negatively curved, or flat. Simply stated, the "Flatness Problem" is that an extremely large time has already passed and the expansion is still going on, but not too vigorously. This suggests that the universe in which we live is rather close to being flat.

The standard way of describing this is that

$$\Omega \equiv \rho/\rho_{cr} = (8\pi/3)\mathrm{G}\rho/\mathrm{H}^2 = 1, \qquad (1)$$

where H is the value of the Hubble constant which describes the rate of expansion of the universe. Now the value of Ω changes in time and its present value can be measured with great difficulty yielding $0.1 < \Omega_0 < 10$ which seems like a horrendous range. But as Robert Dicke (1969) first pointed out, even this range requires that Ω must have been much nearer unity in the past. The problem arises in just how much closer to unity Ω must have been.

While the structure of the early phases of the universe are complicated by inflation and general relativity, the vast majority of the dynamical history can be understood in terms of rather classical Newtonian physics. Following Dicky[3] and using simple Newtonian mechanics one gets

$$\frac{1}{2}\rho \mathrm{v}^2 - 4\pi \mathrm{G}\rho^2 \mathrm{r}^3/3 = \mathrm{E} = \Psi + \Phi \qquad (2)$$

where Ψ and Φ are the kinetic and gravitational binding energy per unit volume respectively. E is the total energy density. Since both Ψ and Φ were far larger in magnitude in the past while E remains constant, the relative value of say $\mathrm{E}/|\Phi|$ must become smaller and smaller in the past. One can choose most any epoch in the early history. If one takes $0.1 < \Omega_0 < 10$ to be representative of the present conditions, then at the decoupling of matter and radiation at about a million years $\Omega \gg 1 \pm 0.0001$ and $(\mathrm{E}/|\Phi|) < 0.0001$. This represents a remarkable balance between the gravitational binding energy and the energy of expansion. To extend the argument through the earlier era to the first three minutes when nuclear physics dominated the structure requires that the agreement between Φ and $|\Psi|$ must have been good to one part in 10^{16}. Application of inflation pushes the agreement to absurdly high values ranging between 10^{43} to 10^{55} in Alan Guth's original exposition of the inflation cosmology[5]. The value is simply determined by when one chooses to begin the modeling of the universe. The earlier the time, the closer the agreement between Ψ and Φ and the nearer $(\mathrm{E}/|\Psi|)$ must be to zero. There is an end to this progression. When one inquires into the structure of the early universe, the physical equations fail at a time known as the Planck time $\mathrm{t_p} = 5.4 \times 10^{-44}$ sec. Lightman and Gingerich[8] place the required agreement at this time to be about 1 part in 10^{59}. In other words $(\mathrm{E}/|\Psi|)) < 10^{-59}$.

So far there is no real problem for cosmology since the above argument simply notes the instability in the dynamical equations of motion whether they arise from Newtonian mechanics, General Relativity, or during the inflation era. The problem arises when one suggests that a number so small must really represents an initial condition on the universe and it is tasteless to suggest that this initial condition

is anything other than $(E/|\Psi|) = 0$. If E was ever zero it is forever zero and $\Omega_0 = 1$. It is generally considered a triumph of the Inflation Cosmology that it not only solves the horizon problem, but that by *reductio ad absurdum* it appears to have eliminated the "Flatness Problem". However, it is clear that any cosmology that pushed the structure of the universe back to the Planck time would have reached the same conclusion.

So compelling is this argument that it has transcended speculation and achieved dogma. The real "Flatness Problem" arises when one places the statement $\Omega_0 = 1$ in confrontation with the observations. Time and again careful measurement of the visible matter, and the influence of unseen baryonic matter on the dynamical behavior of the visible matter, has let thoughtful observers to conclude that $\Omega_0 \sim 0.3$ and is probably closer to 0.1. Analysis of the chemical composition of the universe compared with careful models based on nuclear physics which generate such a composition also suggested an $\Omega_0 \ll 1$. Additional problems arise with the dynamical age of the universe compared with the age of the oldest stars should $\Omega_0 = 1$. This has led some to suggest a cosmological constant and the balance (90%) of the matter is comprised of non-baryonic particles whose existence has yet to be detected. The consequences of $\Omega_0 = 1$ seem extreme enough that we should re-examine the foundation of the argument.

While the quantitative view of the "Flatness Problem" is usually characterized by the ratio of the mean density of the universe to the value required to stop its expansion (i.e. Ω), we note that neither is a directly observed quantity. Indeed, Peebles[11] devotes an entire chapter to methods of determining the current mass density from observable parameters which are basically the total mass within some known volume of the universe. There are incredible difficulties in determining the actual mass of material and one must also be careful in choosing the volume. Should the volume be too large there will be systematic variations throughout the volume resulting from the expansion of the universe itself. If the volume is too small, it will not contain a representative sample of the mass of the universe and the result will be biased. Indeed, as we find increasingly large structures in the universe, it is not obvious that such a volume exists, but will leave it to others to explore the implications of this possibility.

Although the quantity Ω is usually expressed in terms of relative densities, it is also equal to the ratio of the observed mass within the suitable chosen volume to the mass required to stop the expansion of the material within that volume. Since these are the observed quantities we shall focus on them in formulating the flatness problem. This form also makes the balance between the gravitational binding energy and the expansion energy more transparent. In the case where the universe is asymptotically balanced, the expansion energy exactly equals the gravitational binding energy so that the total energy

is zero. Thus a comparison between the actual mass in the carefully chosen volume and the mass required to stop the expansion is a comparison between the actual gravitational binding energy and the expansion energy that would lead to a barely closed universe.

That we can choose such a volume and ignore the remainder of the universe can be traced back to Newton who showed that a spherically symmetric distribution of matter surrounding a spherical volume played no role in determining the gravitational field within that volume. There is a parallel theorem known as Birkhoff's Theorem[9] that plays the same role in General Relativity. Thus the expansion dynamics of the universe can be deduced from the motions and matter within our specially chosen volume and such an argument can be made at any epoch in the history of the universe.

In order to demonstrate the impact of evolving small perturbations or uncertainties in the initial conditions of the universe to the present epoch, let us consider the impact of Heisenberg's Uncertainty Principle on the mass contained within a small volume at the earliest time at which we may still expect that physics as we know it to describe the universe – the Planck time. If that mass is m_i then

$$\delta(mc^2)\delta t = \hbar \tag{3}$$

A reasonable value for δt is the Planck time itself so that

$$\delta m_i = \hbar/t_p c^2 = 2m_p, \tag{4}$$

where m_p is the Planck mass $(\hbar c/4G)^{\frac{1}{2}}$. An alternate way of viewing this uncertainty is to write

$$\delta p \delta x = \delta(m_i c)\delta x = \hbar, \tag{5}$$

which leads to the same result

$$\delta m_i = \hbar/\ell_p c = 2m_p, \tag{6}$$

so that the location of m_i is uncertain by the ultimate smallest length ℓ_p beyond which we might expect the very equations of physics to fail. While this may be an uncomfortably early time for some, it is the only reasonable time at which to investigate and apply an initial condition since it represents the ultimate initial time. Although the application of initial conditions in general relativity is tricky, there can be little doubt that some form of quantum theory must apply to them. Even though the proper reconciliation of quantum theory with general relativity has yet to be formulated, it is not unreasonable to expect a fundamental property of the universe expressed by the Heisenberg Uncertainty Principle to hold as one approaches the point where those initial conditions apply. If it does not, then the

very concept of an initial condition is unlikely to have any meaning and all consequences of an initial condition having a particular value are suspect.

If we then take this as the uncertainty in the mass that will evolve into the mass used to determine the observed value of W_0, then the initial uncertainty in W_i at the Planck time will be

$$(d\Omega)_i = 2m_p/m_0 > 2m_p/m_c, \tag{7}$$

where m_0 is the present mass within the volume used to measure the current mass density ρ_0. This is because we have taken the mass m_0 within the specially selected volume to be conserved by definition when shrinking the volume back to the earliest allowed time t_p. In order to make this as small an uncertainty as possible we take the density to be the closure density $\rho_c = (3H_0^2/8\pi G)$ and the corresponding mass to be the closure mass m_c so that the uncertainty in the initial value of Ω_i becomes

$$(\delta\Omega)_i > 2m_p/(4\pi R_0^3\rho_c/3) = 2Gm_p/(3H_0^2R_0^3). \tag{8}$$

Here H_0 is the current value for the Hubble constant. If we take the radius of the volume used to determine the current mass density and the associated closure density to be 1000 Mpc then

$$(\delta\Omega)_i \geq 4.71 \times 10^{-60}/h^2, \tag{9}$$

where h is the current Hubble constant in units of 100 km sec^{-1} Mpc^{-1}. Thus we see if $h < 1$, the uncertainty in $(\delta\Omega)_i$ is likely to be of the order of 10^{-59}. However, that is just the inverse of the amplification factor (e.g. Lightman and Gingerich[8]) which $\delta\Omega$ will experience in evolving to the present. Thus we could expect the initial indeterminacy of the initial value of Ω_i to evolve into an uncertainty in Ω_0 of

$$(\delta\Omega)_0 \cong 1. \tag{10}$$

It is worth noting that choosing R_0 in equation (8) to represent a smaller volume increases $(\delta\Omega)_i$, which weakens the flatness constraint even further. The same result applies for using a smaller matter density such as the very uncertain observed value of the matter density ρ_0. If one goes to the other extreme and chooses the largest possible volume for determining ρ_0, then one could take $(R_0 = c/H_0)$ to be the size of the visible universe and one gets that

$$(\delta\Omega)_i > t_pH_0 \cong 10^{-60}, \tag{11}$$

which suggests that

$$(\delta\Omega)_0 > 0.1. \tag{12}$$

However, choosing such a large volume to determine the mean density would include systematic and model dependent corrections introducing far larger uncertainties in Ω_0 than those resulting from the amplified uncertainty of the initial conditions.

Thus it would seem that if the foundations of quantum mechanics are applicable during the very earliest phases of the universe, initial conditions related to present day observables should have tiny uncertainties which have evolved into significant uncertainties today. This does not mean that the contemporary value of Ω_0 cannot be measured to greater precision than that suggested by the value of $\delta\Omega_0$. It does suggest that such a measured value cannot place limits on the initial conditions of the universe that exceed those estimated by equation (10). The thrust of this argument is that it is unreasonable to expect any initial value of a dynamical constraint on the evolution of the universe to be known with absolute precision and that plausible uncertainties in that initial value lead to results which are today totally compatible with observation. Thus one cannot use the so-called "Flatness Problem" to justify the existence of non-baryonic matter or non-zero values of the cosmological constant since such just ifications require the absolute equality of the expansion energy and the gravitational binding energy (i.e. $\Omega_0 = 1.00... \pm 0.00...$) .

While a value of Ω_0 on the order of a few tenths must still be regarded as remarkable, it is not magical or mystical in any way. The observed value also has the benefit of reducing some of the current difficulties in reconciling the dynamical age of the universe with the age of its oldest constituents. One can still have the philosophically satisfying result that the total energy of the universe is as close to zero as one could expect it to be without being forced to the extreme conclusions compelled by it being identically zero. The argument simply says that the universe is as well behaved as quantum mechanics will allow it to be. But perfection at any era is unattainable, and conclusions which require perfection should be regarded as suspect.

Since the mass of the observable universe increases steadily in time, it is a fair question to ask if it will ever become large enough for the quantum uncertainty in Ω_i to be small enough to allow for a measurement of Ω_0 which would place stringent enough limits on the initial conditions to justify perfection. From the expression for $(\delta\Omega)_\mathrm{i}$ obtained for the largest observable mass that could be used for obtaining its minimum value, it is clear that the smallest expected value decreases linearly with the age of the universe. However, as Dicke[3] shows, the expansion of a zero pressure universe characteristic of its late stages grows as $\mathrm{t}^{2/3}$. Thus the gravitational binding energy will approach zero as $\mathrm{t}^{-2/3}$ implying the amplification factor of the initial uncertainty will continue to grow as $\mathrm{t}^{2/3}$. This would suggest that

if one could use the entire mass of the visible universe to determine the actual mean density of the universe, its difference from the value required to exactly close the universe would approach zero as the cube root of the Hubble age of the universe. This would appear to hold out some hope for those classicist who seek perfection between the expansion energy and the gravitational binding energy. Since an asymptotically flat universe contains an arbitrary amount of matter, the time will come when the mass of the visible universe becomes arbitrarily large. Since the uncertainty in the initial condition for the observable universe remains the Planck mass, the initial uncertainty will become arbitrarily small. Fortunately the dynamical growth of the uncertainty in the observable mass grows slower than the initial value decreases so the idyllic result of an exact balance in the initial condition for the entire observable universe that can eventually exist can be realized. The unfortunate aspect of this argument is that a measurement which could have convincing accuracy lies a thousand times the current Hubble age in the future.

References

1. Alpher, R.A. and Herman, R. Nature, 162, pp. 774, (1948).

2. Alpher, R.A. and Herman, R. In *Modern Cosmology in Retrospective*, (ed. B.Bertotti, R.Balbinot, S. Bergia, and A. Messina) Cambridge University Press, Cambridge, pp.137-140, (1990).

3. Dicke, R.H. *Gravitation and the Universe*, Jayne Lectures for 1969, American Philosophical Society, Independence Square, Philadelphia, pp. 55-60, (1970).

4. Gamow, G. Nature, 162, pp 680, (1948).

5. Guth, A. Phys. Rev. D., 23, pp. 347-356, (1981).

6. Hubble, E. Proc. of the Natl. Acad. Sci. 15, pp. 168, (1929),

7. Hubble, E. *The Realm of the Nebula*, Yale University Press, New Haven, pp. 102- 123, (1936).

8. Lightman, A., and Gingerich, O. Science 255, pp.690-691, (1992).

9. Misner, C. W., Thorne, K.S., and Wheeler, J.A., *Gravitation*, W.H.Freeman, San Francisco, pp. 843-844, (1973).

10. Osterbrock, D.O. In *Modern Cosmology in Retrospective*, (ed. B.Bertotti, R.Balbinot, S. Bergia, and A. Messina) Cambridge University Press, Cambridge, pp. 258-275, (1990).

11. Peebles, P.J.E. *Physical Cosmology*, Princeton University Press, Princeton N.J. pp. 56-115, (1971).

12. Penzias, A.A., and Wilson, R.W. Ap.J. 142, pp. 419, (1965).

13. Weinberg, S. *The First Three Minutes*, Basic Books, Inc. New York, (1977).